An Application of Latin Squares Methodology in Design Optimization

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The method of latin squares was applied to a three-dimensional mission space for a typical ground attack mission profile and, independently, to a three-dimensional design space to select input conditions for an aircraft sizing program. For each mission, the output takeoff gross weights (TOGW) and corresponding design points determined a quadratic polynomial for TOGW in the three design variables by least squares regression analysis. The minimum TOGW's corresponding to each of the 13 selected mission points were then determined and these were fit by a quadratic polynomial in the three mission variables. The result is a simple algebraic relationship between mission requirements and TOGW for geometrically optimized fighter aircraft. In a second application of these methods, two engine variables were included in the design space.

Nomenclature

AR	= aspect ratio, independent design variable
BPR	= engine bypass ratio, independent design variable
DLN	= landing distance, dependent variable, ft
DTO	= takeoff distance, dependent variable, ft
GSS	= sustained normal load factor, *g*
MACH	= dash Mach number
OPR	= overall engine pressure ratio, independent design variable
RNG	= dash range, n. mi.
STR	= internal expendable payload, independent mission variable, lb
TAC	= acceleration time, min
TOGW	= takeoff gross weight, dependent mission variable, lb
TW	= aircraft thrust-to-weight ratio, independent design variable
WOS	= aircraft wing loading, independent design variable, lb/ft^2

I. Introduction

THIS study demonstrates the applicability of statistical and optimization techniques to the task of relating minimum takeoff gross weight to mission variables for fighter aircraft. This process is seen as an essential first step in the identification of the most cost-effective configuration for performing a particular task in a given scenario. It is expected that the techniques applied here to aircraft sizing will be equally applicable to aircraft costing and effectiveness studies if adequate cost and effectiveness models can be developed. The possibility then exists for analytically investigating the relationship between cost-effectiveness and mission variables.

The purpose of the effort was to demonstrate the application of latin squares, regression analysis, and optimization methods to determine the relationship between the takeoff gross weight and the mission requirements for an aircraft optimized in terms of its design geometry to yield the minimum gross weight required to perform the mission.

The following nine step approach was used in this study:

1) A mission profile was selected for simulation.

2) Three independent mission variables were selected and the range of their values defined. This three-dimensional space was called the "mission space."

3) Three independent design variables were selected and their range of values defined. This three-dimensional space was called the "design space."

4) The simple latin squares method was used to select several particular mission space points at which minimum weight designs were to be determined for points within the design space.

5) The simple latin squares method was used to select several particular design space points at which aircraft sizing was performed.

6) An aircraft sizing program was used to determine the takeoff gross weight (TOGW), takeoff distance (DTO), and landing distance (DLN) for each selected design point at each selected mission point.

7) For each mission point, TOGW was related to the design parameters by fitting a quadratic expression in the three design parameters to the sizing data using a regression analysis program, SURFIT.

8) Using these expressions, the minimum TOGW for each mission was determined with an optimization program, OAPEN.

9) Finally, SURFIT was used to obtain the desired relationship between these minimum TOGW's and their associated mission parameters.

In a second application of these methods, two engine parameters, OPR (overall pressure ratio) and BPR (bypass ratio) were added to the design space, and minimum TOGW's were determined for a single mission point subject to a succession of increasingly severe constraints on DTO, DLN, GSS (sustained normal load factor in *g*), and TAC (acceleration time.) Additional details regarding this study can be found in Ref. 1.

II. Mission Point and Design Point Selection

For the complete mission profile described in Fig. 1, the three parameters MACH (dash Mach number), RNG (dash range), and STR (internal expendable payload weight) were

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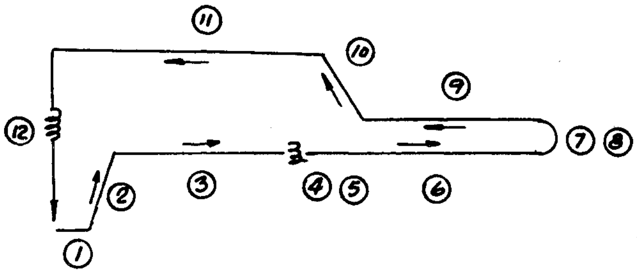

Fig. 1 Mission profile: 1) Start engines, taxi, takeoff. 2) Climb at Mach .85 to 35,000 ft. 3) Outbound cruise – 275 n.mi., Mach .85, 35,000 ft. 4) Loiter – 30 min, Mach .85, 35,000 ft. 5) Accelerate to dash Mach no. (MACH). 6) Outbound dash at MACH. 7) Combat – one max-*g* turn at MACH. 8) Drop stores. 9) Inbound dash at Mach. 10) Climb to 45,000 ft using afterburner. 11) Inbound cruise – 325 n.mi., Mach .85, 45,000 ft. 12) Descend to sea level, loiter for 20 min. at Mach .4, land.

permitted to vary over the range specified in Table 1. Within this "mission space," 27 (3^3) mission points were identified by the maximum, mean, and minimum values of each parameter. The simple latin-squares method was used to select several points for further study from these candidate points.

The aircraft and engine design parameters selected to be varied were WOS (wing loading), AR (aspect ratio), TW (aircraft thrust to weight ratio), OPR, and BPR. The five-dimensional "design space" in which their variation was constrained is described in Table 2. The design variables were fixed at five equally spaced values for a total of 5^5 (3125) candidate design points for each of the 3^3 (27) mission points. The task of determining aircraft TOGW's at all possible combinations ($3^3 \times 5^5 = 84{,}375$) would have been formidable. The simple latin squares selction method was used to reduce this number to a manageable 675 (15 mission points × 45 design points).

III. Simple Latin Squares Method

A statistical selection technique known as the simple latin squares method was used to logically identify representative subsets of the complete mission space and the complete design space. The method is based on random numbers, field algebra, and the algebra of integers, and yields a sequence of values for each variable which is formed by joining together permutations of the values of that variable. Hence, each variable is stepped through all of its values every k data points, where k is the number of values each variable is allowed to assume. The values of the variables are normalized on the interval $(-1, 1)$. For n independent variables, there are n matrices established to generate the design points or mission points.[2] The method identifies $2p^2-p$ points from the available candidates, where p is the smallest prime number in the set $\{p \geq n\}$.

Table 1 Mission space

Variable	Range of values
MACH	1.2 – 1.6
RNG	200 – 400 n. mi.
STR	5000 – 10,000 lb

Table 2 Design space

Variable	Range of values
OPR	10 – 30
BPR	0.2 – 2.2
WOS	80 – 160 lb/ft^2
TW	0.6 – 1.0
AR	1.5 – 3.5

The 15 selected mission points are depicted in Fig. 2. It should be noted that missions 8 and 9 are duplicates, as are missions 12 and 15. Reference 3 describes an *orthogonal* latin squares method for which the selected points are always unique. Application of the simple latin squares method to the five-dimensional design space produced the 45 unique points presented in Table 3.

The latin squares method, simple or othogonal, is probably not the best method for point selection. What is needed is a technique for choosing the best few points to fairly (with equal weighting) represent each subdomain of the entire space. Other "point scattering" techniques have been tried and found to produce satisfactory results.[4] Of these the D-Optimal method shows considerable promise.[5]

IV. Aircraft Sizing

The study was completed in two parts using two different aircraft sizing programs furnished by the Air Force Flight Dynamics Laboratory Design Branch (AFFDL/FXB) to simulate the mission of Fig. 1. In Part A, a scalable fixed-cycle engine was used to reduce the design space to three dimensions (WOS, AR, TW). Consequently, the optimum TOGW relations were independent of OPR and BPR. In Part B a single mission point was selected for analysis of the full five-dimensional design space. For the inclusion of OPR and BPR in the sizing program it was necessary for AFFDL/FXB personnel to develop 25 engine decks.

The 12-segment mission shown in Fig. 1 is potentially representative of future fighter aircraft designs. The specified mission was to be performed with a one-man crew, with all expendable stores carried internally. The aircraft was to make one 360-deg turn at the dash Mach number and altitude before expending the internal stores. Distance credit was given for all mission segments except subsonic loiter, combat, store expenditure, and prelanding loiter. In Part A, the dash range (RNG) was the total distance covered in the outbound and inbound dash segments. In Part B, RNG was fixed at 250 n. mi.

V. Surface Fit Approximation

In order to apply mathematical optimization methods to the aircraft designs generated by the sizing program, the dependent variables TOGW, DTO, and DLN were represented as analytical functions of the independent design variables WOS, AR, and TW. These "surface fit approximations" for the dependent variables were obtained through regression analysis using the computer program SURFIT (SURface FIT) supplied by the Air Force Aero Propulsion Laboratory (AFAPL). SURFIT was developed by Mc-

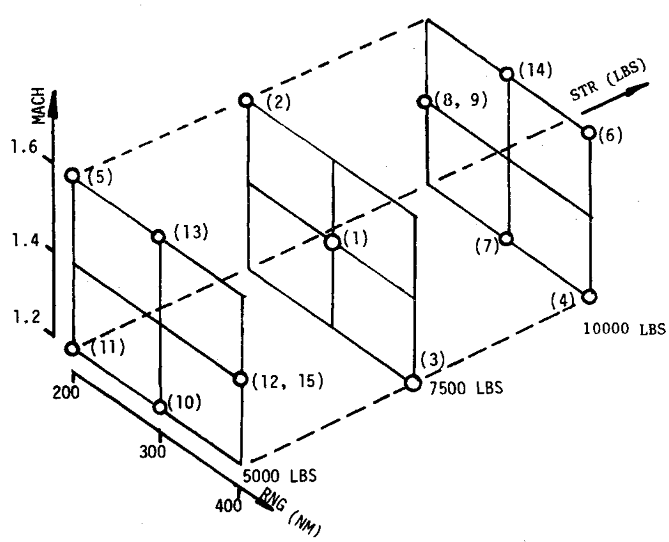

Fig. 2 Latin squares selected mission points.

Table 3 Latin squares selected design points

Case no.	WOS (lb/ft^2)	AR	TW	OPR	BPR
1	120	2.5	.8	20	1.2
2	140	3.5	.6	15	1.2
3	160	2.0	.9	10	1.2
4	80	3.0	.7	30	1.2
5	100	1.5	1.0	25	1.2
6	160	3.0	.7	10	1.7
7	100	3.5	.6	25	2.2
8	140	1.5	1.0	15	.2
9	80	2.0	.9	30	.7
10	140	3.0	.9	25	1.7
11	160	1.5	.7	20	1.7
12	80	2.5	1.0	15	1.7
13	100	3.5	.8	10	1.7
14	120	2.0	.6	30	1.7
15	80	3.5	.8	15	1.7
16	120	1.5	.7	30	2.2
17	160	2.0	.6	20	.2
18	100	2.5	1.0	10	.7
19	160	3.5	1.0	30	2.2
20	80	2.0	.8	25	2.2
21	100	3.0	.6	20	2.2
22	120	1.5	.9	15	2.2
23	140	2.5	.7	10	2.2
24	100	1.5	.9	20	1.7
25	140	2.0	.8	10	2.2
26	80	2.5	.7	25	.2
27	120	3.0	.6	15	.7
28	80	1.5	.6	10	.2
29	100	2.5	.9	30	.2
30	120	3.5	.7	25	.2
31	140	2.0	1.0	20	.2
32	160	3.0	.8	15	.2
33	120	2.0	1.0	25	1.7
34	160	2.5	.9	15	2.2
35	100	3.0	.8	30	.2
36	140	3.5	.7	20	.7
37	100	2.0	.7	15	.7
38	120	3.0	1.0	10	.7
39	140	1.5	.8	30	.7
40	160	2.5	.6	25	.7
41	80	3.5	.9	20	.7
42	140	2.5	.6	30	1.7
43	80	3.0	1.0	20	2.2
44	120	3.5	.9	10	.2
45	160	1.5	.8	25	.7

Donnell-Douglas Corporation, McDonnell Aircraft Company, St. Louis, Missouri.[6]

Although other options were available in SURFIT, it was decided that the dependent variables TOGW, DTO, and DLN would be represented by second order polynominals of the form

$$\begin{aligned}\mathrm{TOGW} = {} & A_0 + A_1(\mathrm{WOS}) + A_2(\mathrm{AR}) + A_3(\mathrm{TW}) + A_{11}(\mathrm{WOS})^2 \\ & + A_{12}(\mathrm{WOS})(\mathrm{AR}) + A_{13}(\mathrm{WOS})(\mathrm{TW}) + A_{22}(\mathrm{AR})^2 \\ & + A_{23}(\mathrm{AR})(\mathrm{TW}) + A_{33}(\mathrm{TW})^2 \end{aligned} \tag{1}$$

In general summation notation for n independent variables, Eq. (1) can be written

$$\mathrm{TOGW} = A_0 + \sum_{i=1}^{n} A_i X_i + \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ij} X_i X_j \tag{2}$$

where the A's are the coefficients and the X's are the independent variables. There were 10 coefficients for Part A and 21 coefficients for Part B.

Typical of the surface fits obtained for all missions in Part A are those shown in Eqs. (3-5) for mission 13. As is characteristic of surface fits developed by regression analysis, some of the 10 terms were insignificant and do not appear.

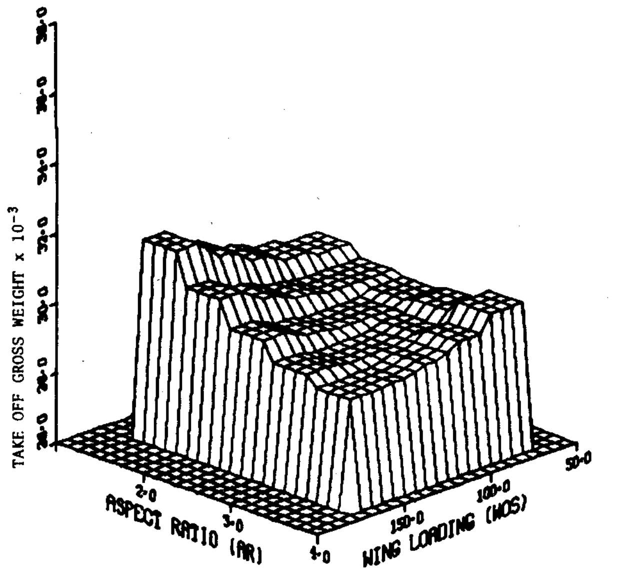

Fig. 3 Three-dimensional plot of TOGW vs WOS and AR for mission 13 (TW = 0.6).

$$\begin{aligned}\mathrm{TOGW} = {} & 38850.416 - 28156.2033(\mathrm{TW}) + .1780(\mathrm{WOS})^2 \\ & - 16.0080(\mathrm{WOS})(\mathrm{AR}) + 450.7710(\mathrm{AR}) \\ & - 1591.2582(\mathrm{AR})(\mathrm{TW}) \\ & + 26021.8945(\mathrm{TW})^2 \end{aligned} \tag{3}$$

$$\begin{aligned}\mathrm{DTO} = {} & 3239.8422 + 48.9005(\mathrm{WOS}) - 7296.6377(\mathrm{TW}) \\ & - .4827(\mathrm{WOS})(\mathrm{AR}) - 30.1409(\mathrm{WOS})(\mathrm{TW}) \\ & + 4549.4329(\mathrm{TW})^2 \end{aligned} \tag{4}$$

$$\begin{aligned}\mathrm{DLN} = {} & -954.8010 + 36.4208(\mathrm{WOS}) + 2556.2243(\mathrm{TW}) \\ & - .0238(\mathrm{WOS})^2 + 1.1496(\mathrm{WOS})(\mathrm{AR}) - 39.7089(\mathrm{AR})^2 \\ & + 118.2842(\mathrm{AR})(\mathrm{TW}) - 1649.1797(\mathrm{TW})^2 \end{aligned} \tag{5}$$

Three dimensional plots of TOGW vs TW and AR were generated from the surface fit approximations in order to provide a check on the predicted minimum TOGW output from the optimizer. The plot presented in Fig. 3 is for mission 13 at TW = .6 and was generated from Eq. (3). While not suitable for determining numerical values of TOGW, these plots were useful for visualizing qualitative functional dependence. The variety of points from which a three dimensional surface can be viewed makes it imperative that the reader note the axis orientation for each plot. The three dimensional plots were generated on the CALCOMP Plotter using DISSPLA.[7]

VI. Optimization

The optimization problems considered in this study required the minimization of a performance function such as TOGW subject to the inequality constraint that another performance function such as DTO be equal to or less than some specified value. An additional "box constraint" was imposed such that the independent variables remained inside the design space. An object deck of the computer program OAPEN (Optimization Analysis by PENalty function) was supplied by the Air Force Aero Propulsion Laboratory (AFAPL) and was used to accomplish the various optimizations in this study. OAPEN was developed by The Beoing Aerospace Company, Seattle, Washington under contract F33615-73-C-2084 to AFAPL.[8]

The two optimizations performed for each mission were to: 1) minimize TOGW with no constraints on DTO and DLN other than they must be positive; and 2) minimize TOGW

Table 4 Summary of minimum TOGW results for unconstrained and constrained ( ) optimization[a] for Part A

	Mission			Optimum design points											
No.	Mach	RNG	STR	TOGW	(TOGW)	WOS	(WOS)	AR	(AR)	TW	(TW)	DTO	(DTO)	DLN	(DLN)
1	1.4	300	7500	27,320	27,554	141.79	111.55	3.5	2.98	.6	.6	4293	3506	5318	4237
2	1.6	200	7500	29,871	29,991	120.27	102.27	3.15	2.94	.6	.6	4036	3506	4787	4109
3	1.2	400	7500	25,669	25,758	137.21	120.58	3.35	3.06	.6	.6	3904	3503	4917	4353
4	1.2	400	10,000	30,074	30,238	144.77	120.55	3.5	3.01	.6	.6	4086	3504	5260	4422
5	1.6	200	5000	26,127	26,127	111.58	103.01	3.5	3.5	.6	.6	3750	3500	4179	3866
6	1.6	400	10,000	42,182	43,335	160.00	122.41	3.5	3.5	.69	.711	4595	3506	5816	4503
7	1.2	300	10,000	28,999	29,154	144.28	120.22	3.5	3.07	.6	.6	4073	3493	5352	4503
8	1.4	200	10,000	29,982	30,298	145.10	111.52	3.5	2.95	.6	6	4382	3506	5728	4459
10	1.2	300	5000	20,371	20,395	134.66	120.90	3.49	3.27	.6	.6	3834	3502	4836	4364
11	1.2	200	5000	19,736	19,749	130.43	121.06	3.5	3.37	.6	.6	3729	3501	4816	4490
12	1.4	400	5000	25,437	25,523	129.73	111.53	3.24	3.00	.6	.6	3983	3505	4493	3892
13	1.6	300	5000	29,035	29,335	157.42	116.36	3.5	3.29	.648	.679	4779	3507	5567	4210
14	1.6	300	10,000	36,289	37,360	149.38	102.15	3.5	2.74	.6	.6	4851	3512	5813	4052

[a]Constraints: DTO $\leq$ 3500 feet and DLN $\leq$ 4500 feet.

subject to the constraints that DTO must be equal to or less than 3500 ft and DLN must be equal to or less than 4500 ft. The independent variables WOS, AR, and TW were "box constrained" to not be outside the range of values noted in Table 2 for the design space.

OAPEN is designed to find an optimum design parameter vector using surface fit functions to approximate the true functions of the design parameters in an optimal design problem. In this study, the general optimization problem to be solved can be written as

$$\text{Minimize} \quad f_1(X)$$
$$\text{Subject to} \quad f_j(X) \leq c_j \qquad (j=2,\ m) \tag{6}$$

where X is the vector of independent variables, the f's are the performance functions approximated by surface fits, and the c's are the values of the upper limits for the constraint functions. OAPEN solves this problem by the penalty function method in which the inequalities of Eq. (6) are used to establish a penalized cost function of the form

$$F(X) = f_1(X) + P_K \sum_{j=2}^{m} (\mathrm{CV}_j)^2 \tag{7}$$

where $f_1(X)$ is as defined for Eq. (6), P_K is a weight factor which modulates the severity of violating the constraints, and CV represents the violation of the constraint's inequalities. P_K corresponds to the allowable tolerance on violation of the constraints. A default value of $P_K = 50$ exists in the program and corresponds to a 2% tolerance. A value of $P_K = 100$ (1% tolerance) was used in this study. CV in Eq. (7) has the form (DTO − 3500) when DTO is constrained to be less than or equal to 3500 feet. CV is equal to zero if the constraint is satisfied and takes on the value (DTO − 3500) when the constraint is violated (exceeded).

For the constrained minimization performed in this study, Eq. (7) can be written

$$F(X) = \mathrm{TOGW} + 100(\mathrm{DTO} - 3500)^2 + 100(\mathrm{DLN} - 4500)^2 \tag{8}$$

OAPEN uses the Fletcher-Reeves conjugate gradient search method to find the minimum $F(X)$ given in Eq. (8). The gradient of $F(X)$ is computed from the TOGW, DTO, and DLN surface fit approximations which are input to the program. The algorithm requires an initial value from which to begin the gradient search. The optimizations performed were based on surface fit approximations for TOGW, DTO, and DLN as quadratic functions of WOS, AR, and TW such as those given in Eqs. (3-5). The mean values of WOS, AR, and TW (120, 2.5, .8) were input as the starting point for the gradient search algorithm.

The results of constrained and unconstrained optimizations are presented in Table 4. In general, these particular constraints were satisfied at a cost of 300-500 lb additional TOGW. Wing loadings (WOS) and aspect ratios (AR) were reduced while thrust-to-weight ratios (TW) remained the same with the exceptions of mission 6 and mission 13 where TW increased. Figure 3 indicates the existence of an apparent minimum TOGW for mission 13 in the general region predicted by OAPEN. The minimum was clearly confirmed by two-dimensional plots of TOGW vs the design variables.

VII. Investigation Results

Part A

The optimum aircraft design results are presented in Table 4 for the thirteen unique missions of Part A. Surface fit approximations were developed to relate TOGW, DTO, and DLN for these optimum aircraft to the three independent mission variables MACH, RNG, and STR. The quadratic polynomial approximations technique discussed earlier and the program SURFIT were used for the regression analysis with the following results for the unconstrained case.

$$\begin{aligned}\mathrm{TOGW} = {} & 97109.316 - 118597.9044(\mathrm{MACH}) \\ & - 144.1725(\mathrm{RNG}) \\ & + 2.6280(\mathrm{STR}) + 42465.2574(\mathrm{MACH})^2 \\ & + 86.9971(\mathrm{MACH})(\mathrm{RNG}) \\ & - .6542(\mathrm{MACH})(\mathrm{STR}) + .0735(\mathrm{RNG})^2\end{aligned} \tag{9}$$

$$\begin{aligned}\mathrm{DTO} = {} & 1159.6500 + 10.6167(\mathrm{RNG}) + .0770(\mathrm{STR}) \\ & + 4.3653(\mathrm{MACH})(\mathrm{RNG}) - .0266(\mathrm{RNG})^2\end{aligned} \tag{10}$$

$$\begin{aligned}\mathrm{DLN} = {} & 3756.6445 + .1771(\mathrm{STR}) - 1246.7666(\mathrm{MACH})^2 \\ & + 13.2589(\mathrm{MACH})(\mathrm{RNG}) - .0311(\mathrm{RNG})^2\end{aligned} \tag{11}$$

Some statistical analysis of each equation is presented in Table 5.

To help visualize the relationships given by Eqs. (9-11), three-dimensional plots of TOGW, DTO, and DLN for optimized aircraft with 5000 lb of stores were generated using these equations for various values of MACH and RNG. Presented in Fig. 4-6, these plots dramatically depict trends over the allowable variation of MACH and RNG. The TOGW plot (Fig. 4) is particularly effective as a means of locating an apparent minimum or regions deserving more detailed analysis. For example, it can be seen from Fig. 4 that

Table 5 Multiple correlation and maximum error for Part A

Variable	Multiple correlation	Maximum % error
TOGW	.99825	−2.63
DTO	.89380	−7.19
DLN	.90604	−9.82

an apparent minimum exists for MACH = 1.2 in the vicinity of RNG = 275 n. mi and not at RNG = 200 n. mi., the minimum range in the mission space. This result contradicts the input data from Table 4 where the minimum TOGW did occur for MACH = 1.2 at RNG = 200 n. mi. (mission 11). If Eq. (9) was taken as exact, then it can be shown that the minimum TOGW of 19,775 lb occurs for RNG = 270.58 n. mi. For MACH = 1.2 and RNG = 200 n. mi., Eq. (9) predicts TOGW = 20,140 lb. The difference of 365 lb is, however, attributed to a buildup of error in the surface fits leading to Eq. (9). The surface slope discontinuities of Figs. 4-6 and the viewpoint make it impossible to determine specific values of TOGW, DTO, or DLN from these slots.

Part B

In Part B of the study the same methods discussed above were applied to the case of a fixed mission of the same basic profile shown in Fig. 1. The mission parameters were fixed as follows: MACH = 1.95, RNG = 250 n. mi., STR = 5500 lb. The variation of the five independent design variables is given in Table 2 with the exception that the maximum WOS was reduced to 120 lb/ft^2. The simple latin squares method selected 45 design points from the 5^5 candidate points determined by 5 equally spaced values for each of the 5 variables. For each of these 45 design points and the mission profile, TOGW, DTO, DLN, TAC, and GSS were computed by an AFFDL/FXB aircraft sizing program. The program required new engine tables (thrust and fuel flow vs Mach number and altitude) for each of the 25 combinations of OPR and BPR.

SURFIT was applied to sizing results to determine analytic expressions relating TOGW, DTO, DLN, GSS and TAC to the five independent design variables.

$$\begin{aligned}\mathrm{TOGW} = {} & 45849.08 + 53.31(\mathrm{OPR})^2 - 3988.15(\mathrm{OPR})(\mathrm{TW}) \\ & + 86838.3926(\mathrm{TW})^2 - 900.9884(\mathrm{AR})\end{aligned} \tag{12}$$

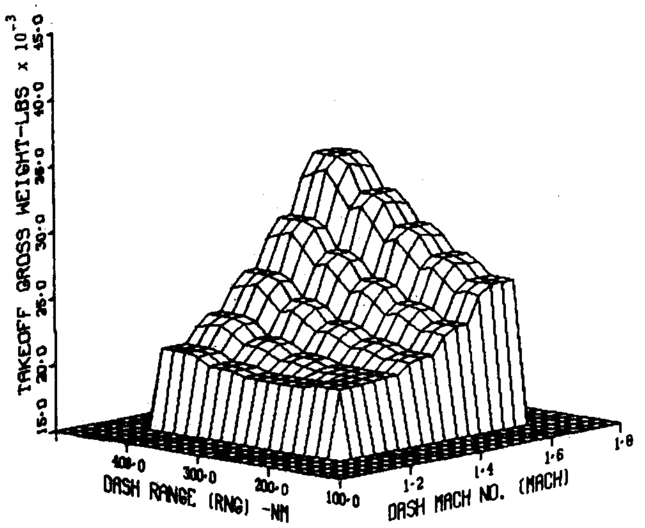

Fig. 4 Three-dimensional plot of minimum TOGW vs MACH and RNG (STR = 5000 lb).

$$\begin{aligned}\mathrm{DTO} = {} & 9781.7277 + 77.6748(\mathrm{WOS}) - 14341.0841(\mathrm{TW}) \\ & - 3086.5461(\mathrm{AR}) - 41.3483(\mathrm{WOS}) \\ & - 7.4961(\mathrm{WOS})(\mathrm{AR}) + 6227.6061(\mathrm{TW})^2 \\ & + 1605.3901(\mathrm{TW})(\mathrm{AR}) + 333.0961(\mathrm{AR})^2\end{aligned} \tag{13}$$

$$\begin{aligned}\mathrm{DLN} = {} & 2848.4882 + 71.6588(\mathrm{WOS}) - 1881.9125(\mathrm{AR}) \\ & - 36.3198(\mathrm{WOS})(\mathrm{TW}) - 5.4750(\mathrm{WOS})(\mathrm{AR}) \\ & + 570.6408(\mathrm{TW})(\mathrm{AR}) + 245.5962(\mathrm{AR})^2\end{aligned} \tag{14}$$

$$\begin{aligned}\mathrm{TAC} = {} & 4.9253 - .1225(\mathrm{BPR}) - .008399(\mathrm{WOS}) \\ & - 6.9671(\mathrm{TW}) + .000013336(\mathrm{OPR})^2 \\ & + .002236(\mathrm{BPR})^2 \\ & + .08645(\mathrm{BPR})(\mathrm{TW}) + .00003341(\mathrm{WOS})^2 \\ & + .003342(\mathrm{WOS})(\mathrm{TW}) - .0008496(\mathrm{WOS})(\mathrm{AR}) \\ & + 3.0129(\mathrm{TW})^2 + .0159(\mathrm{AR})^2\end{aligned} \tag{15}$$

$$\begin{aligned}\mathrm{GSS} = {} & .6559232 + .8677(\mathrm{AR}) - .00588(\mathrm{WOS})(\mathrm{AR}) \\ & + .3618(\mathrm{TW})(\mathrm{AR})\end{aligned} \tag{16}$$

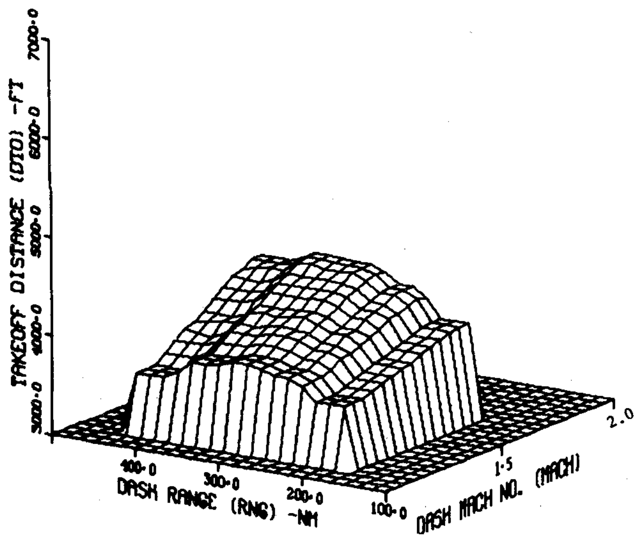

Fig. 5 Three-dimensional plot of DTO vs Mach and RNG (STR = 5000 lb).

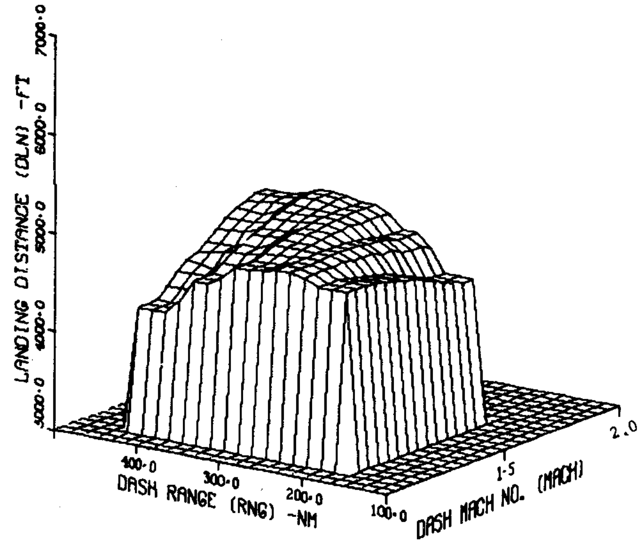

Fig. 6 Three-dimensional plot of DLN vs MACH and RNG (STR = 5000 lb).

Table 6 Multiple correlation and maximum error for Part B

Variable	Multiple correlation	Maximum % error
TOGW	.94306	20.26
DTO	.99626	– 15.08
DLN	.98791	– 13.08
TAC	.99888	– 3.00
GSS	.99508	12.09

Table 7 Minimization results

Variable	Minimization case 1	2	3	4	5
TOGW, lb	39,224	39,678	39,678	39,918	41,514
DTO, ft	2729	3002	3002	2442	1682
DLN, ft	3537	3990	3990	3399	2797
TAC, min	1.35	1.17	1.17	1.17	1.02
GSS, g	2.40	2.08	2.08	2.50	2.99
OPR	22.41	23.55	23.55	24.41	27.66
BPR	1.2	2.2	2.2	2.2	2.2
WOS, lb/ft^2	100	116.94	116.94	97.77	80
TW	.60	.63	.63	.65	.74
AR	3.5	3.5	3.5	3.5	3.5
Constraints:					
DTO	None	≤ 3000	≤ 3000	≤ 3000	≤ 3000
DLN	None	None	≤ 4000	≤ 3500	≤ 3000
TAC	None	≤ 1.17	≤ 1.17	≤ 1.17	≤ 1.17
GSS	None	None	≥ 2.0	≥ 2.5	≥ 3.0

As a result of the regression analysis, WOS and BPR terms do not appear in Eq. (12) for TOGW, nor do OPR and BPR terms appear in Eqs. (13), (14), or (16) for DTO, DLN, or GSS.

The multiple correlation and maximum error for Eqs. (12-16) are presented in Table 6. Although the maximum error in TOGW was 20.26%, there were only 7 of 45 data points at which the error was greater than 10%, with most errors less than 7%. For DTO, the maximum error was 15.08%, although there were only four points with errors greater than 4%. DLN had a maximum error of 13.08%, but there were only four cases where the error exceeded 5%. GSS had a maximum error of 12.09% and there were only two other cases where the error exceeded 4%. These error may be attributed to the fact that TOGW varied between 40,591 and 99,643 lb for the 45 design points. The simple polynomial surface fit approximation cannot adequately represent this large range of data. At this point the conscientious designer should redefine the design space and repeat the analysis to reduce errors.

Based on Eqs. (12-16), several minimizations with a variety of constraints were performed using OAPEN. The results are presented in Table 7.

These results indicate that there is only 2290 lb (5.8%) difference in TOGW between case 1 (39,224 lb) where there were no constraints, and case 5 (41,514 lb), where the most severe constraints were applied. The optimum design configurations were similar for all constrained cases in that there was very little change in OPR, BPR, TW, and AR. Note that BPR, TW, and AR were either on or very near the boundary of the design space. Wing loading (WOS) was very sensitive to the constraints on landing distance (DLN) and acceleration time (TAC).

The variations of TOGW with OPR, AR, and TW for the unconstrained minimum case are shown on the three dimensional plots presented in Fig. 7-9. These plots also indicate that the minimum TOGW probably lies outside the design space and that higher aspect ratios (AR) and lower thrust-to-weight ratios (TW) should be considered for the unconstrained case. Also, higher bypass ratios (BPR) should

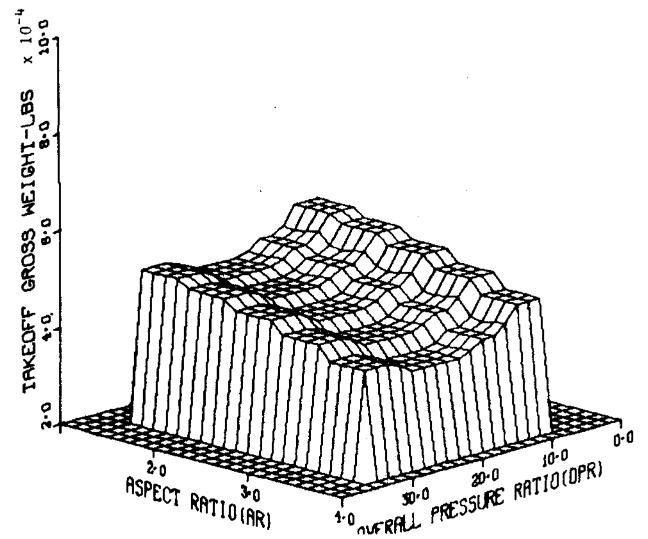

Fig. 7 Three-dimensional plot of minimum TOGW vs OPR and AR from optimized aircraft (TW = 0.6).

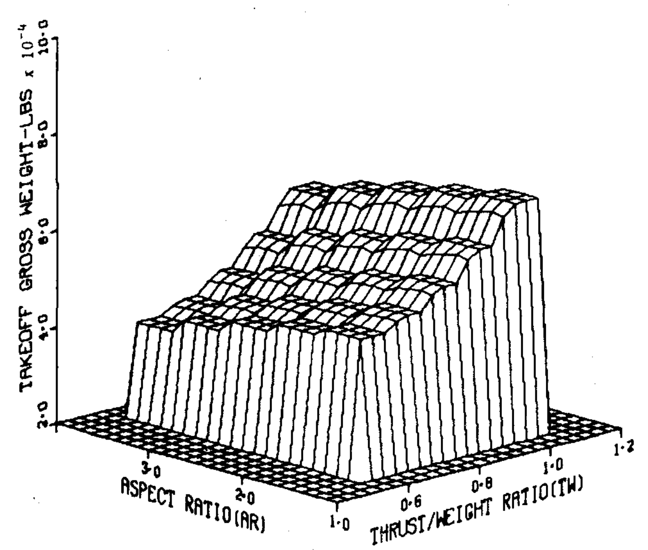

Fig. 8 Three-dimensional plot of TOGW vs TW and AR for optimized aircraft (OPR = 22.41).

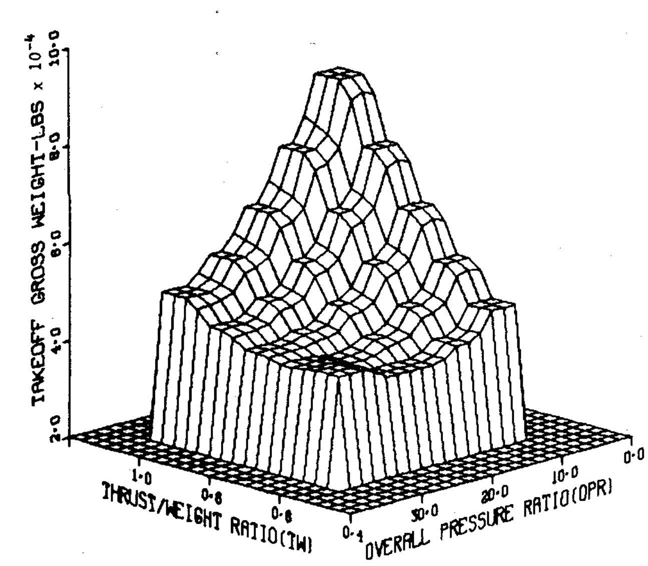

Fig. 9 Three-dimensional plot of TOGW vs TW and OPR for optimized aircraft (AR = 3.5).

be considered, particularly if the constraints in cases 2-5 become overriding requirements. Note that as TW requirements increase, as in case 5, the optimum OPR also increases and may eventually exceed the design space limit of 30.

VIII. Conclusions

The latin squares, surface fit, and optimization methodologies are very useful, flexible, and time saving tools for design and mission analysis. As more and more variables are included in such analyses, the need for these methods increases dramatically. Results from a single application of these methods may indicate the need for a second iteration with new limits (extended, limited, or shifted) on the design/mission variables. It is believed that the separate treatment of design and mission spaces used here yields more accurate results than if the mission and design variables had been all included together in a single "design and mission space." However, the current approach did require a threefold increase in sizing points over the other approach.

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